* Electricity *

Electricity is the set of physical phenomena associated with the presence of motion of matter that has a property of electric charge.

The phenomena associated with the electric charges at rest is called static electricity or electrostatics.

The phenomena associated with the charges in motion is called current electricity.

* Electric Charge :-

It is an intrinsic property of some elementary particles that gives raise an interaction between them and consequently to the host of material phenomena described as electrical.

Electric charge is a scalar quantity. It's SI unit is Coulomb (C). A proton has a positive charge (+e), and an electron has a negative charge (−e), where

$$e = 1.6 \times 10^{-19} \text{ Coulomb}$$

Electric Current :- The flow of electric charges through conductor in particular direction is called electric current.

Electric current in a conductor across an area held perpendicular to the direction of flow of charge is defined as the amount of charge flowing across that area per unit time.

If Q is the charge that flows across the given area in time t, then the current strength is

$$\boxed{I = Q/t} \quad \text{——— (i)}$$

This relation is hold good only for the uniform flow of charge through conductor.

If charge ΔQ passes through an area in time t to $t+\Delta t$, then the current I is given by

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \quad \text{——— (ii)}$$

This is called instantaneous current.

* The SI unit of electric current is ampere (A).

If one Coulomb of charge crosses an area in one second uniformly, then the current through that area is one ampere.

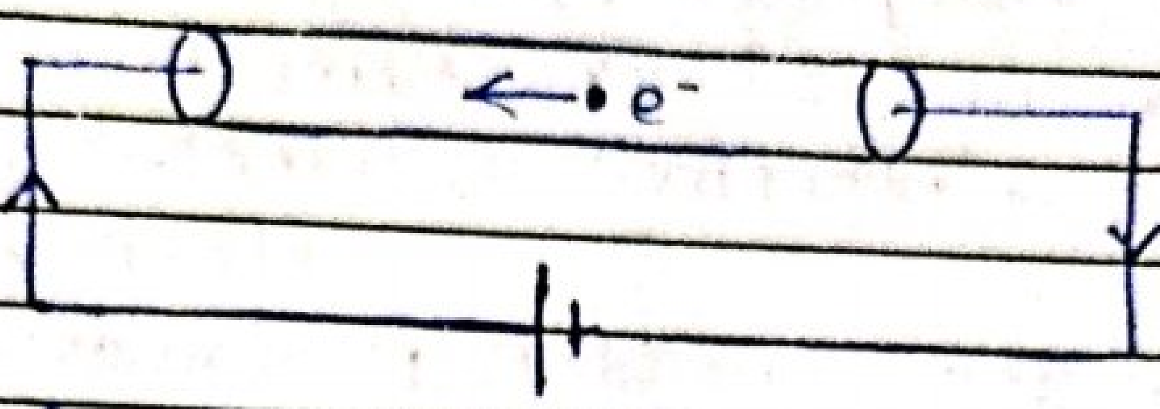
$$1 \text{ ampere} = \frac{1 \text{ Coulomb}}{1 \text{ second}}$$

* Electric current is a scalar quantity, although electric current has both magnitude and direction, because the laws of ordinary algebra are used to add electric currents and the laws of vector addition are not applicable to their addition.

* Conventional Current –

The direction of flow of positive charge for convention in a conductor gives the direction of current, called conventional current.

The direction of flow of electrons gives the direction of electronic current.



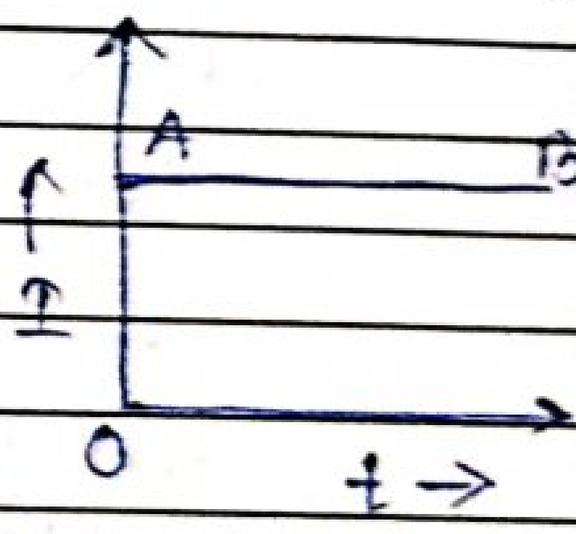
The direction of electronic current is opposite to that of conventional current.

Types of electric current

The electric current can be classified into the following catagories –

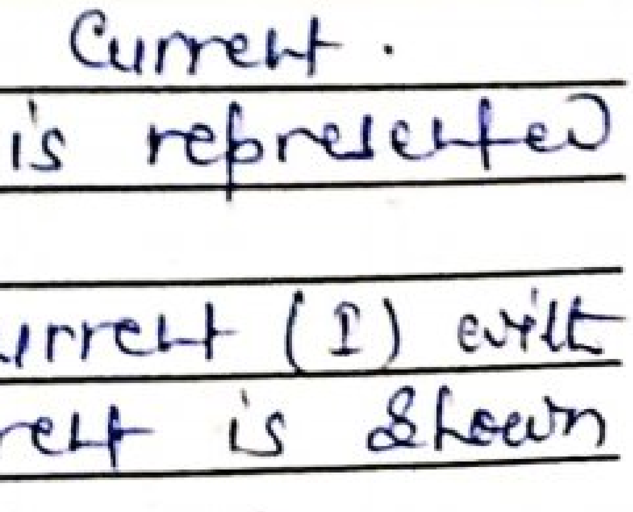
(i) Steady current :→ The current whose magnitude does not change with time is called steady current.

The variation between current (I) and time (t) for steady current is represented by a straight line AB in adjoining figure.



(ii) Varying current :– The current whose magnitude changes with time is called varying current.

The varying current is represented by a curve of different nature is shown in adjoining figure.

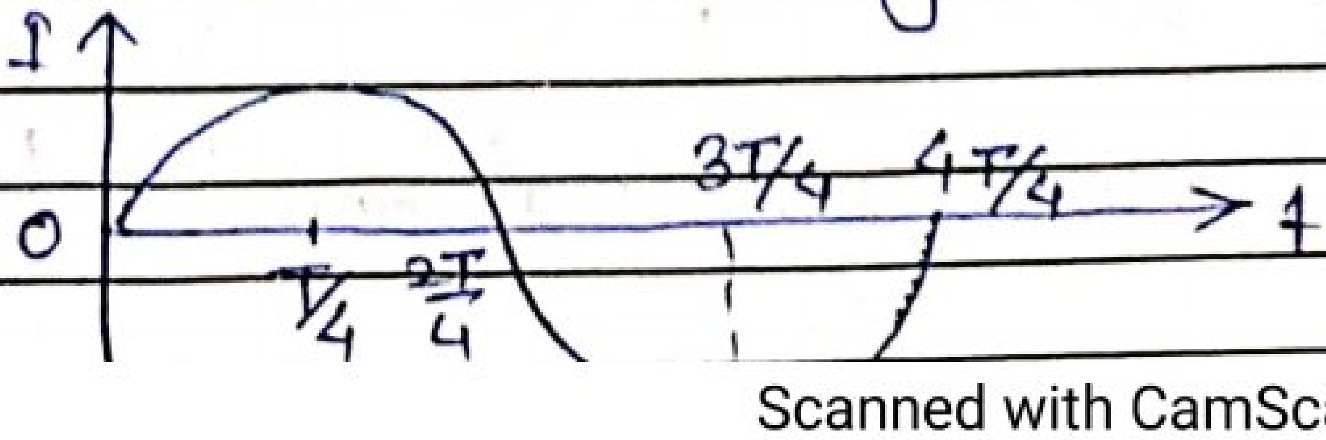


(iii) Alternating current :–

The current whose magnitude changes continuously with time and direction changes periodically is called alternating current.

Alternating current is represented by a sine curve or cosine curve.

The variation of current (I) with time (t) for sinusoidal alternating current is shown in figure



Drift Velocity:- The average velocity with which free electrons get drifted towards the positive end of the conductor under the influence of an external electric field is called drift velocity of the free electrons.

Drift velocity of the electrons is of the order of 10^{-5} m/s.

Expression: —

Every metal has large number of free electrons or conduction electrons which are in a state of continuous, rapid zig-zag motion in the conductor. The number of free electrons per cubic meter of the conductor will be of the order of 10^{29} and the average thermal speed of the free electrons in random motion is of the order of 10^{5} m/s at room temperature, so the average thermal velocity of the electrons is zero.

If $\vec{u_1}, \vec{u_2}, \vec{u_3} \ldots \vec{u_n}$ are random thermal velocities of n electrons in a metal, then the average thermal velocity of electrons is

$$\vec{u} = \frac{\vec{u_1} + \vec{u_2} + \cdots + \vec{u_n}}{n} = 0 \quad \text{—— (I)}$$

Again, when potential difference V is applied across the two ends of metallic conductor of length 'l' then electric field $E = V/l$ is setup inside the conductor and the free electron in the conductor experiences a force, and free electrons are accelerated towards higher potential. On moving they suffer frequent collisions against the ions and lose energy, so the electrons acquire a small velocity towards the positive end of the conductor. This average velocity is called drift velocity.

Suppose e is the electronic charge and m is the mass of each electron then the force experience by each electron in electric field is

$$\vec{F} = -e\vec{E} = m\vec{a}$$

$$\therefore \boxed{\vec{a} = \frac{-e\vec{E}}{m}} \quad \text{—— (II)}$$

Let $\vec{v_1}, \vec{v_2}, \ldots \vec{v_n}$ are the velocities of n electrons after the last collision then the average final velocity of the free electrons is the drift velocity of the electrons and is

$$\vec{v_d} = \frac{\vec{v_1} + \vec{v_2} + \cdots + \vec{v_n}}{n}$$

$$= \frac{(\vec{u_1} + \vec{a}\tau_1) + (\vec{u_2} + \vec{a}\tau_2) + \cdots + (\vec{u_n} + \vec{a}\tau_n)}{n}$$

$$= \left(\frac{\vec{u_1} + \vec{u_2} + \cdots + \vec{u_n}}{n}\right) + \vec{a}\left(\frac{\tau_1 + \tau_2 + \cdots \tau_n}{n}\right)$$

$$= 0 + \vec{a}\tau$$

Where $\tau = \frac{\tau_1 + \tau_2 + \cdots \tau_n}{n}$ is the average relaxation time and its value is of the order of 10^{-14} second.

$$\therefore \boxed{\vec{v_d} = \frac{-e\vec{E}}{m}\tau} \quad \text{--- (iii)}$$

Note:- Relaxation time = $\frac{\text{Mean free path for electron}}{\text{Drift velocity of electron}}$

Relation between drift velocity and electric Current :—

Consider a conducting wire of length l and of uniform area of cross-section A, consists of n free electrons per unit volume.

Now, the total number of free electrons in the conducting wire is

$$N = n \times \text{volume of wire}$$
$$= n \times Al$$

If e is the charge on each electron, then total charge on all the free electrons in the conductor

$$q = Ne$$
$$= nAle$$

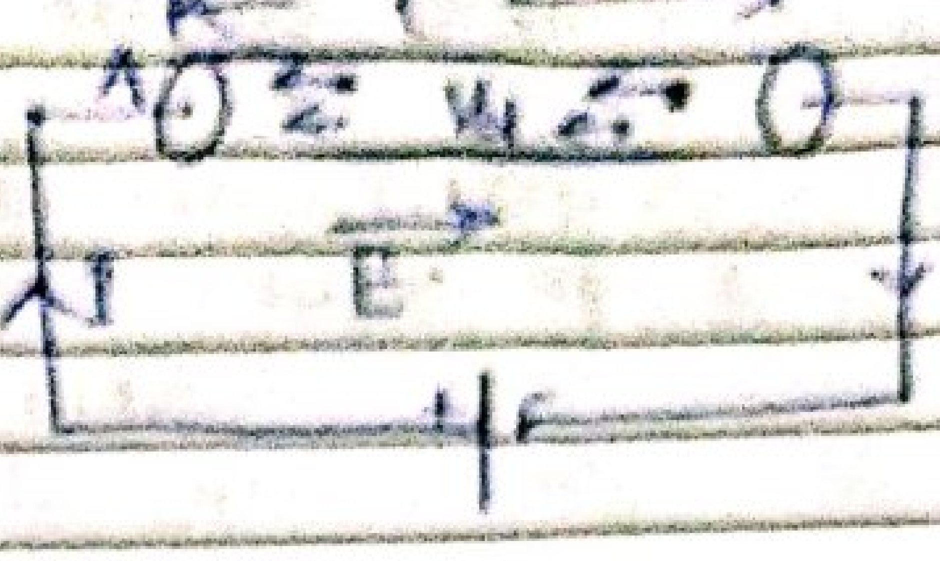
Let a constant potential difference V be applied across the ends of conducting wire, then the electric field setup across the conductor is given by

$$E = V/l$$

Due to this field, the free electrons present in the conductor will begin to move with the drift velocity v_d towards the higher potential.

The time taken by the free electrons to cross the conductor is

$$t = l/v_d$$



Therefore, current in a conducting wire

$$I = q/t$$

$$= \frac{Alne}{l/v_d}$$

$$\boxed{I = v_d A n e}$$

Since, A, n and e are constants

$$\boxed{I \propto v_d}$$

Thus the current flowing through a conductor is directly proportional to the drift velocity.

Note:- The small value of drift velocity ($\sim 10^{-5}$ m/s) produces a large amount of electric current, due to presence of extremely large number of free electrons in a conductor. The propagation of current is almost at the speed of light and involves electromagnetic process. It is due to this reason that the electric bulb glows immediately when switch is on.

Current density :→ Current density at a point in a conductor is defined as the amount of current flowing per unit area of the conductor around that point provided the area is held in a direction normal to the current. It is denoted by J.

Let I be the current distributed uniformly across a conductor of cross-sectional area A. The magnitude of the current density for all points on that cross-section of the conductor is

$$\boxed{J = \frac{I}{A}}$$

We know,

$$I = v_d A n e$$

$$\therefore \boxed{J = \frac{I}{A} = v_d n e}$$

It is a vector quantity. Its direction is the direction of motion of positive charge. The unit of current density is ampere (meter)$^{-2}$ or Am^{-2}.

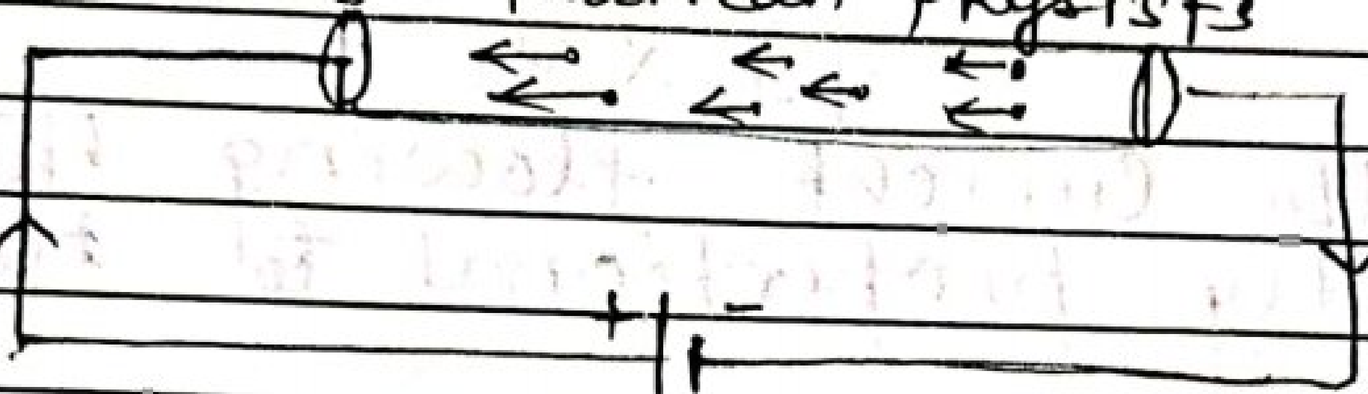
Carriers of electric Current :—

The charged particles which by flowing in definite direction setup an electric current are called current carriers.

The different types of current carriers are as follows—

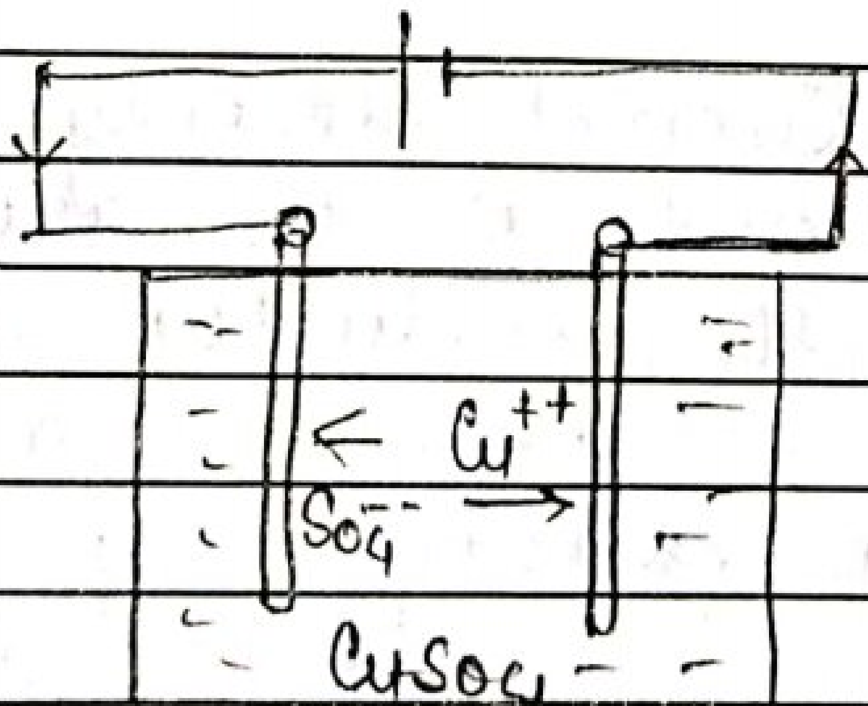
1. In Solids:— In solid conductors, like metals, the valance electrons of the atoms donot remain attached to individual atoms but are free to move throughout the volume of the conductor. Under the effect of an external electric field, the valence electrons move in a definite direction causing electric current in the conductors.

Hence the valance electrons are the current carriers in solid conductors. "Was first experimentally confirmed by American physicists Tolman and Stewart in 1917."



2. In liquid:— In electrolytic liquids, there are positively and negatively charged ions. These ions are forced to move in definite direction under the action of effect of an external electric field, causing electric current.

Thus in liquids, the positively and negatively charged ions are the current carriers.



3. In gases— Ordinary gases are bad conductor of electricity. But they can be ionized by applying high potential difference at low pressure. The ionized gas contains positive ions and electrons. Thus positive ions and electrons are the current carriers in gases.

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* Ohm's law *

The relation between electric current and electric potential difference between two ends of metallic conductor was derived by German Physicist George Simon Ohm in 1828. This relationship is known as Ohm's law and can be stated as follows–

"The current flowing through a conductor is directly proportional to the potential difference applied across its ends, provided the temperature and other physical conditions remain constant."

Thus,

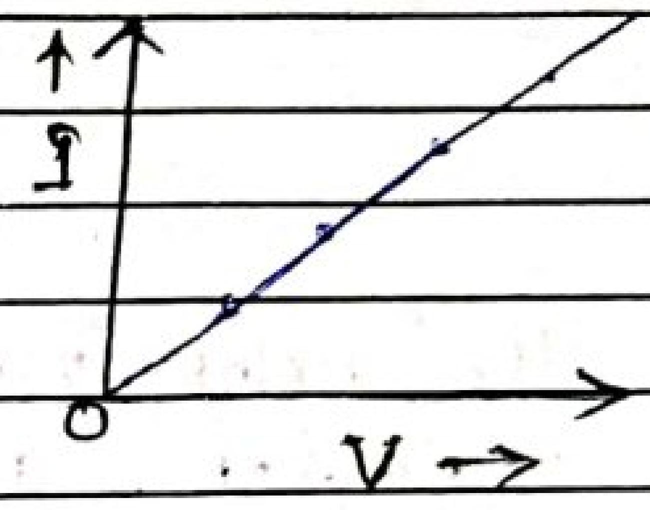
$$I \propto V$$

$$\text{or, } V = RI$$

The proportionality constant R is called resistance of the conductor.

$$\therefore \frac{V}{I} = R$$

The graph between potential difference V applied across a conductor to the current flowing through it is a straight line.



* Derivation of ohm's law *

Consider a conducting wire of length l, area of cross-section A and consists of n free electrons per unit volume. Let potential difference V is applied across the two ends then electric field $E = V/l$ is setup and the free electrons are drifted towards higher potential with the drift velocity

$$V_d = \left(\frac{eV}{ml}\right)\tau \quad \text{—— (i)}$$

The current flowing through wire is given by

$$I = V_d A n e \quad \text{—— (ii)}$$

Substituting the value of V_d in eq. (ii), we get

$$I = \left(\frac{eV}{ml}\tau\right) A n e$$

$$\text{or, } I = \left(\frac{Ane^2\tau}{ml}\right) V$$

$$\text{or, } I = \frac{1}{R} V$$

where $R = \frac{ml}{Ane^2\tau}$ is called resistance of a conductor.

or, $V = RI$

If R is a constant, then

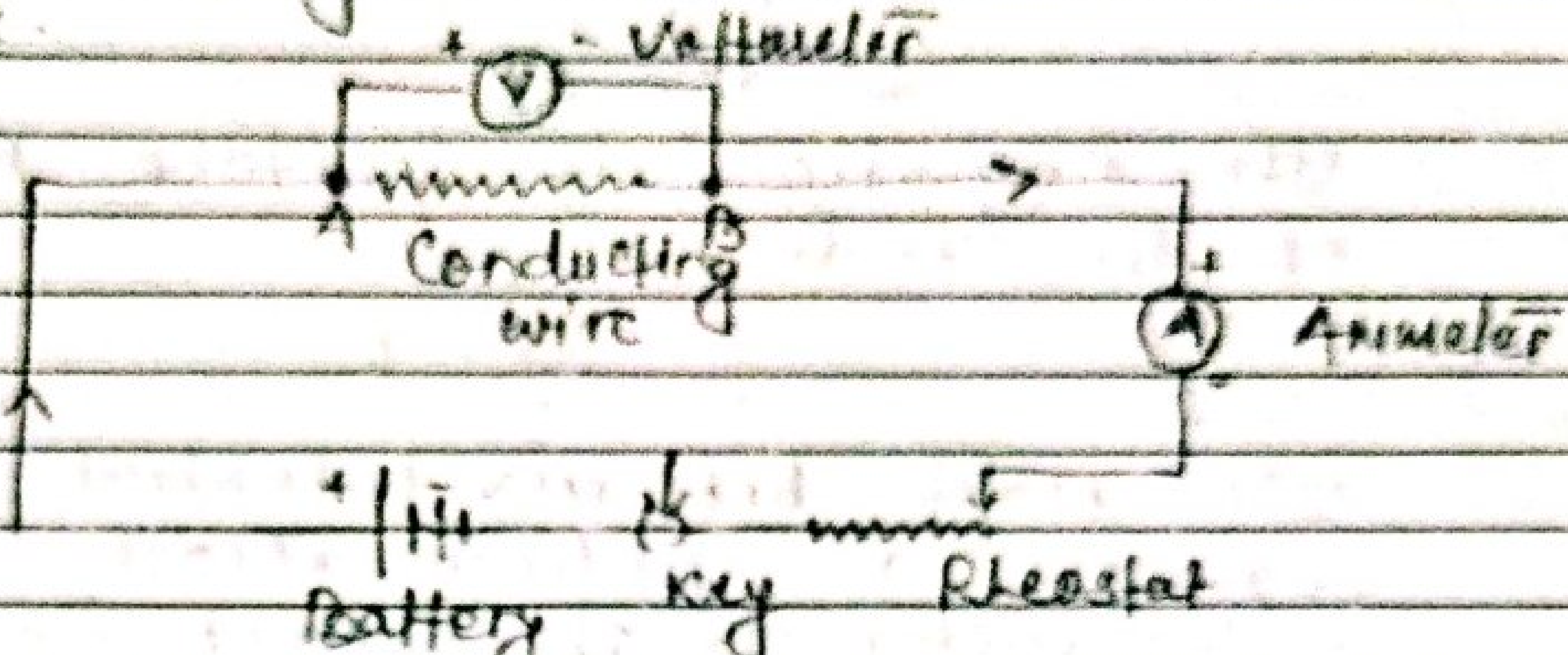
$V \propto I$

or, $\boxed{I \propto V}$

This is ohm's law.

Experimental verification of ohm's law

The experimental arrangement for verifying ohm's law is shown in adjoining figure.



The experimental arrangement consists of a battery, key, rheostat, an ammeter all are connected in series with the conducting wire AB and the voltmeter is connected in parallel with wire.

First of all when key be closed then some current delivered by battery and ammeter and voltmeter both indicate some deflection. The deflection in an ammeter provide current and deflection in a voltmeter measure potential difference across the ends of wire. By adjusting the rheostat, vary the current I through wire and at each value of current measure the corresponding value of potential difference. In each case, the ratio V/I is computed and comes out always constant.

If I_1, I_2 ---- are the currents through wire and V_1, V_2 ---- are the corresponding potential difference, then from experiment

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} = \text{-----}$$

$$\therefore \frac{V}{I} = \text{a constant}$$

$$\therefore V \propto I$$

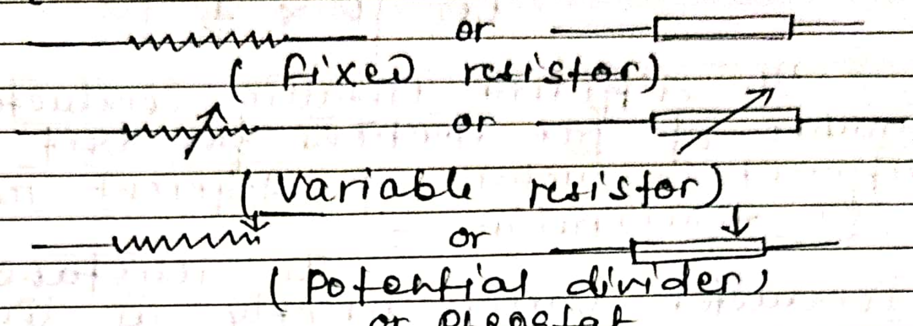
$$\text{or, } I \propto V$$

On plotting the graph, p.d against current, obtain a straight line passing through origin. This verify ohm's law.

The resistance of a conductor is the property by virtue of which it opposes the flow of charge i.e. current through it. Any material that has some resistance is called a resistor.

The resistance is represented by R (for external resistance) and r (for internal resistance).

The symbolic representation of resistance in a circuit is



The resistance of conductor is measured by the ratio of potential difference applied across conductor to the current flowing through it.

Thus.

$$\boxed{R = \frac{V}{I}}$$

The resistance of metallic conductor is

$$\boxed{R = \frac{ml}{Ane^2\tau}}$$

The resistance of conductor is measured in ohm (Ω).

$$\therefore 1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}}.$$

* The resistance of a conductor is said to be 1 ohm if a current of 1 ampere flows through it on applying a potential difference of 1 volt accross its ends. *

The dimensional formula of resistance is

$$R \Rightarrow \frac{[M^1L^2T^{-3}A^{-1}]}{[A^1]}$$

$$\Rightarrow [M^1L^2T^{-3}A^{-2}]$$

Factors on which resistance of metallic conductor depends:

1. On length: — The resistance of a conductor is directly proportional to its length provided area of cross-section and temperature remain constant.

2. On area of Cross-Section:- The resistance of a uniform Conductor varies inversely as area of cross-section provided length and temperature remain constant.

i.e. $\boxed{R \propto \frac{1}{A}}$

3. On nature:- The resistance of Conductor varies inversely as the number of free electrons per unit volume.

i.e. $\boxed{R \propto \frac{1}{n}}$

The different metallic Conductors having different number of free electrons per unit volume as a result different conductors of different resistance.

4. On temperature:-

The resistance of a metallic conductor varies directly as temperature provided length and area of cross section remain constant.

i.e. $\boxed{R \propto \text{temperature} \propto \frac{1}{\tau}}$

Specific resistance or Resistivity

The resistance of a metallic Conductor varies directly as length and inversely as cross-sectional area at constant temperature.

Thus,

$$R \propto l/A$$

$$\text{or, } R = \rho\, l/A$$

Where ρ is a proportionality Constant, Called resistivity or Sp. resistance, whose value depends upon the nature of material of Conductor as well as temperature.

$$\therefore \boxed{\rho = \frac{RA}{l}}$$

But, the resistance of a metallic Conductor is

$$R = \frac{ml}{Ane^2\tau}$$

$$\therefore \boxed{\rho = \frac{m}{ne^2\tau}}$$

If $l = 1$ unit and $A = 1$ square unit, then

$$\boxed{\rho = R}$$

Thus, the resistivity or sp. resistance of a material may be defined by the resistance of a conductor of that material, having unit length and unit area of cross-section.

or, "It is the resistance offered by the unit cube of the material of a conductor."

Note:-

(i) The S.I. unit of resistivity is ohm meter (Ωm).

(ii) Resistivity of material of conductor independent from length and area of cross-section i.e. its size or shape.

(iii) Resistivity depends upon the nature of material of conductor as well as temperature.

Conductance:- The conductance of a conductor is the ease or effort with which electric charges flow through it. It is measured in terms of the reciprocal of resistance.

i.e. $\text{Conductance} = \frac{1}{\text{Resistance}}$

or $\boxed{C \text{ or } G = \frac{1}{R}}$

The S.I. unit of conductance is ohm^{-1} or mho or Siemens (S)

Conductivity:- The reciprocal of the resistivity of a material is called conductivity. It is denoted by σ.

Thus,

$$\text{Conductivity} = \frac{1}{\text{Resistivity}}$$

$$\therefore \boxed{\sigma = \frac{1}{\rho}}$$

The S.I. unit of conductivity is $ohm^{-1} m^{-1}$ or mho m^{-1} or Sm^{-1}.

Relation Connecting J, σ and E (vector form of ohm's law)

Consider a conducting wire of length l, uniform area of cross section A and consists of n free electrons per unit volume. Let potential difference V applied across the ends of wire then electric field is setup in a wire, whose magnitude is

$$E = V/l.$$

The drift velocity of the free electrons in conducting wire is

$$V_d = \frac{eV}{ml}\tau$$

The current flowing through conducting wire is

$$I = V_d A n e$$

or, $\frac{I}{A} = V_d n e$

or, $J = \left(\frac{eV}{ml}\tau\right) n e$ $\quad$ [$\because J = I/A$, current density]

or, $J = \left(\frac{ne^2\tau}{m}\right) E$

or, $J = \frac{1}{\rho} E$ $\quad$ [$\because \rho = \frac{m}{ne^2\tau}$, resistivity of material of wire]

or, $J = \sigma E$ $\quad$ [$\because \sigma = \frac{1}{\rho}$, conductivity of material of wire]

$$\therefore \boxed{\vec{J} = \sigma \vec{E}}$$

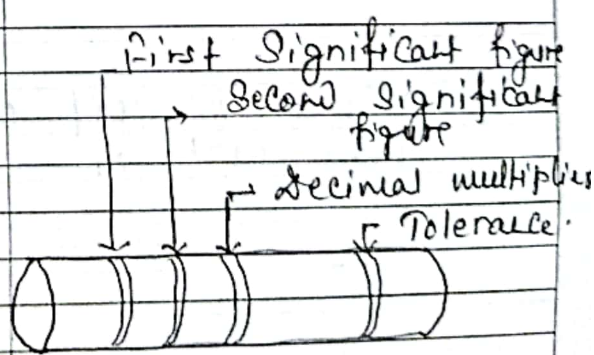
This is the required relation.

Note:- Colour Code for Carbon resistor:-

A colour code is used to indicate the resistance value of carbon resistor and its percentage accuracy. To remember colour code by

B B R O Y of Great Britain had very Good wife
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
0 1 2 3 4 5 6 7 8 9

Colour	letter as an aid to memory	Number	multiplier	Colour	Tolerance
Black	B	0	10^0	Gold	5%
Brown	B	1	10^1	Silver	10%
Red	R	2	10^2	No fourth band	20%
orange	O	3	10^3		
yellow	Y	4	10^4		
Green	G	5	10^5		
Blue	B	6	10^6		
violet	V	7	10^7		
Grey	G	8	10^8		
White	W	9	10^9		



Example:- (1) The colour of the four bands are red, red, red ~~violet~~ silver, the resistance value is

$$R = 22 \times 10^2 \pm 10\%$$

Mobility of charge carriers:-

The mobility of charge carrier is the drift velocity acquired by it in unit electric field. It is denoted by μ.

$$\text{i.e., } \mu = \frac{v_d}{E}$$

As the drift velocity, $v_d = \frac{eE}{m}\tau$

$$\therefore \mu = \frac{eE\tau}{m} \times \frac{1}{E}$$

$$\therefore \boxed{\mu = \frac{e}{m}\tau}$$

For an electron, $\mu_e = \frac{e\tau_e}{m_e}$

And for hole, $\mu_h = \frac{e\tau_h}{m_h}$

The mobilities of both electrons and holes are positive although their drift velocities are opposite to each other.

The S.I. unit of mobility is $m^2V^{-1}s^{-1}$

The practical unit of mobility is $cm^2V^{-1}s^{-1}$

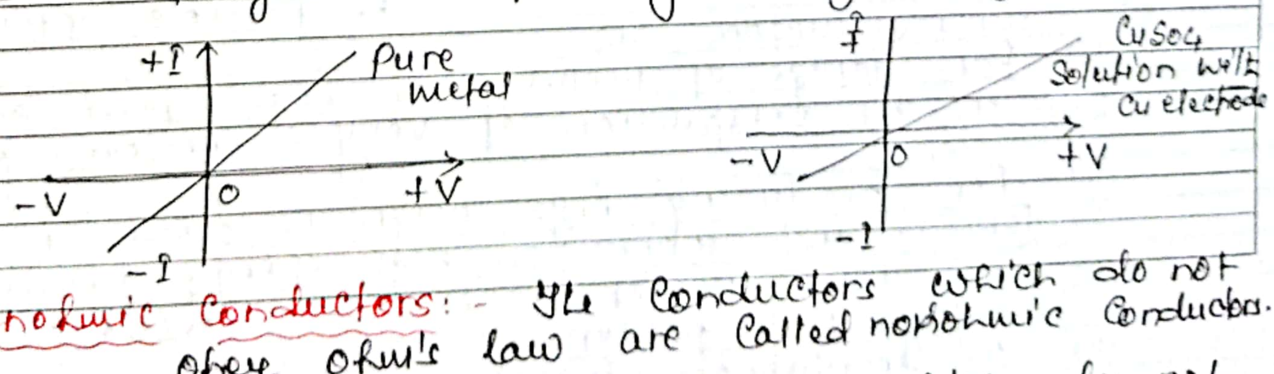
Ohmic Conductors:-

The conductors which obey ohm's law are called ohmic conductors.

The resistance of the conductors which obey ohm's law, called ohmic resistance.

A metallic conductor for small currents and the electrolyte like copper sulphate solution with copper electrodes are ohmic conductors.

In ohmic conductors, the linear relationship between voltage and current ($V \propto I$) holds good. i.e., the V-I graph for ohmic conductors is a straight line passing through origin.



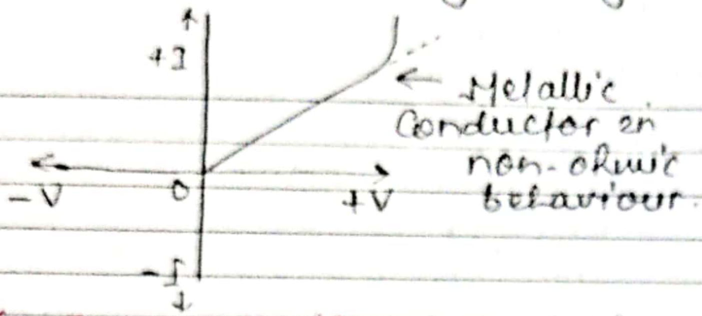
Nonohmic Conductors:-

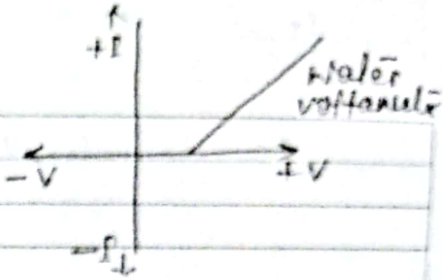
The conductors which do not obey ohm's law are called nonohmic conductors.

The resistance of the conductors which do not obey ohm's law, called non-ohmic resistance.

Metallic conductors at high current, water voltameter, diode, thyristor etc. are the few examples.

The V-I graph for non-ohmic conductors is generally non linear, non unique and straight line does not pass through origin.





Superconductivity. The phenomenon of complete loss of resistivity by certain metals and alloys when they are cooled below a certain temperature is called superconductivity and conductor behaves as superconductor.

The temperature at which a substance undergoes a transition from normal conductor to superconductor in a zero magnetic field is called transition or critical temperature (T_c).

** Superconductivity: In 1911, Prof Kamerlingh Onnes at the university of Leiden (Holland), observed that the resistivity of mercury suddenly drops to zero at a temperature of about 4.2 K and it becomes a super-conductor.

It is belived that near transition temperature, a weak attractive force acts on the electrons which brings them closer to form coupled pairs. Such coupled pairs are not deflected by ionic vibrations and so move without collisions.

Causes of superconductivity. **

Application of Superconductors:- The possible applications of superconductors are.

(i) for storage of memory in high speed computers.
(ii) In the construction of very sensitive galvanometers.
(iii) for producing high magnetic fields required for research work in high energy physics.
(iv) In long power transmission without any wastage of power.
(v) In levitation transportation (trains which move without rails.)

Temperature Coefficient of resistance

The resistance of metallic conductor increases with increase in temperature and decrease in resistance with decrease in temperature.

Let R_0 and R_t are the resistances of a metallic conductor at $0^\circ c$ and $t^\circ c$ respectively. The increase in resistance $(R_t - R_0)$ varies directly as resistance at $0^\circ c$ and rise in temperature.

Thus,

$$R_t - R_0 \propto R_0 \quad \text{---} (i)$$

[∵ Rise in temp' is constant]

and

$$R_t - R_0 \propto (t - 0) \quad \text{---} (ii)$$

[∵ Initial ~~temp~~ resistance is constant]

So,

$$R_t - R_0 \propto R_0 (t-0)$$

or, $R_t - R_0 = \alpha R_0 (t-0)$

Where α is a proportionality constant is called temperature coefficient of resistance.

$$\therefore \boxed{\alpha = \frac{R_t - R_0}{R_0 (t-0)}}$$

Thus, "the temperature coefficient of resistance is defined as the increase in resistance per unit initial resistance per unit degree rise in temperature."

The temperature coefficient of resistance is measured in $^\circ c^{-1}$ or K^{-1}.

<u>Note</u>:- (i) The temperature coefficient of resistance for metallic conductors is positive and of the order of $10^{-3}\ ^\circ c^{-1}$.

(ii) The temperature coefficient of resistance for insulators and semiconductors is negative.

(iii) Alloys like constantan or manganin or eureka are used for making standard resistance coils because the alloys have high resistivity and low temperature coefficient of resistance.

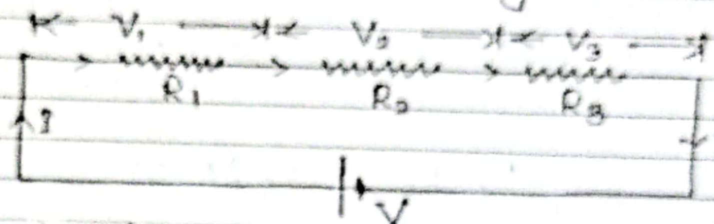
(iv) The materials which have high resistivity and high melting point are used in making heating coils.

(v) The materials which have low resistivity and low melting point are used in making fuse wire (alloy of lead and tin).

Combination of resistances

1. In Series :- If the number of resistances are connected end to end so that the same current flows through each one of them in succession, then they are said to be connected in series.

Consider three resistances R_1, R_2 and R_3 connected in series and combination is connected to the battery of potential difference V then equal current I flows through each resistance.



By ohm's law, the potential drops across the three resistances are

$$V_1 = IR_1 \quad , \quad V_2 = IR_2 \quad , \quad V_3 = IR_3$$

Now, the potential difference across the combination

$$V = V_1 + V_2 + V_3$$

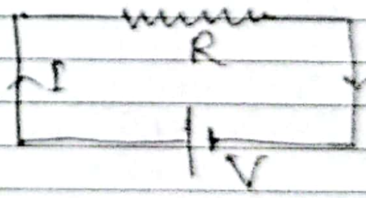
$$= IR_1 + IR_2 + IR_3$$

$$= I[R_1 + R_2 + R_3] \quad \text{——— (i)}$$

If R is a single resistance through which equal current I is flowing across same potential difference V applied, then resistance is said to be equivalent resistance of the three resistances.

So, from ohm's law

$$V = IR \quad \text{——— (ii)}$$



From eqs. (i) and (ii), we get

$$IR = I[R_1 + R_2 + R_3]$$

$$\therefore \boxed{R = R_1 + R_2 + R_3} \quad \longleftarrow \text{(iii)}$$

The equivalent resistance of n resistances connected in series will be

$$\boxed{R_s = R_1 + R_2 + \cdots + R_n} \Rightarrow R_s = \sum_{i=1}^{i=n} R_i$$

Thus when number of resistances are connected in series, their equivalent resistance is equal to the sum of the individual resistances.

Characteristics of series combination :-

(i) current through each resistance is same.

(ii) The total potential drop is equal to the sum of the potential drops across the individual resistances.
(iii) The individual potential drops are directly proportional to individual resistances.
(iv) The equivalent resistance is equal to algebric sum of individual resistances.
(v) Equivalent resistance is larger than the largest individual resistance.

2. In Parallel :–

If number of resistances are connected in between two common points so that each of them provides a separate path for current, then they are said to be connected in parallel.

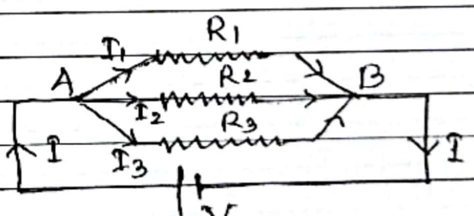
Consider three resistances R_1, R_2 and R_3 connected in parallel between two points A and B. Let V be the potential difference applied across the combination then I_1, I_2 and I_3 currents through the resistances R_1, R_2 and R_3 respectively. So the main current must be

$$I = I_1 + I_2 + I_3$$

Since all the resistances have been connected between the same two points A and B, therefore, potential drop V is same across each of them.
By Ohm's law,

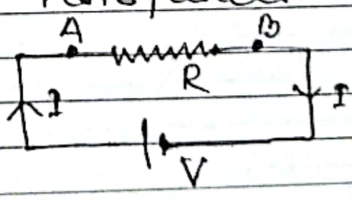
$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

So, $$I = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad \text{——— (i)}$$



If R is a single resistance connected between A and B through which equal current I is flowing across same potential drop, then resistance is said to be equivalent resistance of three resistances. So from Ohm's law

$$I = \frac{V}{R} \quad \text{——— (ii)}$$



From equations (i) and (ii), we get

$$\frac{V}{R} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\therefore \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \quad \text{——— (iii)}$$

The equivalent resistance of n resistances are connected in parallel is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$$\therefore \frac{1}{R_p} = \sum_{i=1}^{i=n} \frac{1}{R_i}$$

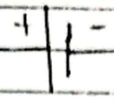
Thus when a number of resistances are connected in parallel, the reciprocal of equivalent resistance of the parallel combination is equal to the sum of the reciprocals of the individual resistances.

Characteristics of Parallel Combination:—

(i) Potential drop across each resistance is same.
(ii) The main current is equal to the currents through individual resistances.
(iii) The current through individual resistance varies inversely as individual resistances.
(iv) The reciprocal of equivalent resistance is equal to the sum of the reciprocals of the individual resistances.
(v) Equivalent resistance is less than the smallest individual resistance.

B.R.Jha

Electric Cell :- It is a device which generate electric energy by the conversion of chemical energy. It is represented by



Electromotive force (EMF) :- The electromotive force of a source may be defined as the work done by the source in taking a unit positive charge from lower to the higher potential. It is denoted by E.

In a battery due to certain chemical reactions, a force of non-electrostatic origin is exerted on the charges of the electrolyte. This force derives positive charges towards positive terminal and negative charges towards negative terminal. As the charges build up on the two terminals, a potential difference is setup between them and hence electric field $\vec{E}$ is setup. This electric field exerts force F_e on charge q in opposite to the direction of force on positive charge $\vec{F_n}$. In the steady state,

$$F_n = F_e$$

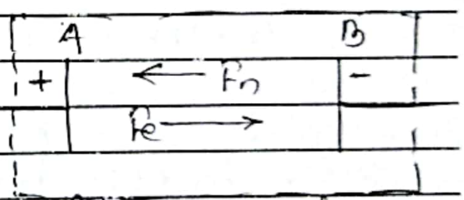


Diagram of a battery.

The work done by non-electrostatic force during the displacement d of a charge q from -ve to positive terminal is

$$W = F_n d$$

So the work done per unit charge is said to be electromotive force

$$E = \frac{W}{q} = \frac{F_n d}{q}$$

Also, if two terminals of a battery are not connected externally, then

$$F_n = F_e = qE$$

$$\text{or, } F_n d = F_e d = qEd = qV$$

Thus

$$E_0 = \frac{Fnd}{q} = \frac{qV}{q}$$

$$\therefore \boxed{E_0 = V}$$

Hence the e.m.f. of a source is the maximum potential difference between it's terminals when it is in the open circuit i.e when it is not sending any current in the circuit.

Also In a closed circuit, the e.m.f. of a source may be defined as the energy supplied by the source in taking a unit positive charge once round the complete circuit.

$$\text{i.e.}\quad E_0 = W/q$$

Terminal Potential difference (P.d.) – The terminal potential difference of a source i.e cell is the potential drop across the two terminals of a cell when a current is being drawn from it. It is denoted by V.

Internal resistance of a Cell:– The resistance offered by the electrolyte of a cell to the flow of current between it's electrodes is called internal resistance of the cell. It is denoted by r.

The internal resistance of a cell depends on following factors –

(i) Nature of the electrolyte.

(ii) It is directly proportional to the distance between the two electrodes.

(iii) It is inversely proportional to the common area of the electrodes immersed in the electrolyte.

(iv) It is directly proportional to the concentration of the electrolyte.

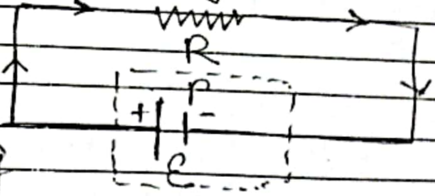
(v) It increases with decrease in temperature of the electrolyte. etc.

Note:– The internal resistance of a freshly prepared cell is usually low but its value increases as more current draw from it.

Relation between internal resistance, e.m.f. and terminal potential difference of a cell—

Consider an electric cell of e.m.f $\mathcal{E}$ and internal resistance r connected to an external resistance R, then constant current I flows through the circuit.

Now, by the definition of e.m.f



Electric cell

$\mathcal{E}$ = (work done in carrying a unit charge from +ve to negative terminal against external resistance R) + (work done in carrying a unit charge from −ve to +ve against internal resistance r)

or, $\mathcal{E} = V + V'$

By ohm's law,

$V = IR$ and $V' = Ir$ (Potential drop across r)

$$\therefore \quad \mathcal{E} = IR + Ir$$
$$= I(R+r)$$

Hence, the current in the circuit is

$$I = \frac{\mathcal{E}}{R+r} \quad \text{——— (i)}$$

The terminal potential difference of a cell that sends current I through the external resistance R is given by

$$V = IR$$
$$= \left(\frac{\mathcal{E}}{R+r}\right)R$$

or, $V(R+r) = \mathcal{E}R$

or, $VR + Vr = \mathcal{E}R$

$$\therefore \quad \boxed{r = \left[\frac{\mathcal{E}-V}{V}\right] \times R} \quad \text{——— (ii)}$$

This is the required relation connecting $\mathcal{E}$, V and R

Note:-

(i) When cell is on open circuit, i.e. $I = 0$, then

$$V = \mathcal{E}$$

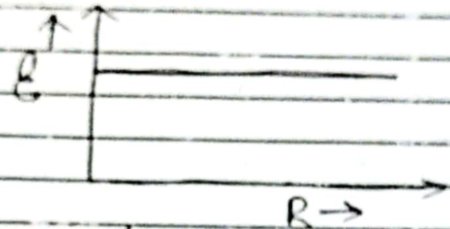
(ii) The terminal p.d. when a current is being drawn from the cell

$$V = \mathcal{E} - Ir$$

(iii) The terminal p.d. when cell is being charged.

$$V = \mathcal{E} + Ir$$

(iv) The graph between e.m.f. and external resistance as no current is drawn from cell is

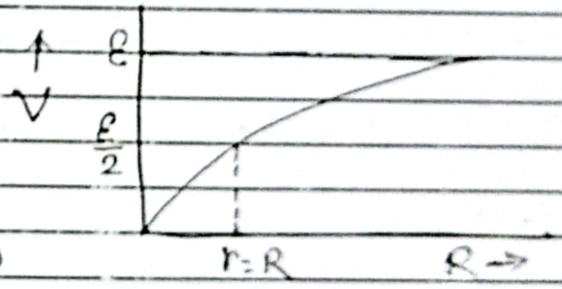


($\mathcal{E}$ vs R graph)

(v) The graph between p.d. and external resistance in a closed circuit is

The terminal p.d. of a cell is

$$V = IR = \left(\frac{\mathcal{E}}{R+r}\right) R$$



(V vs R graph)

As R increases V also increases.

When $R \to 0$, $V = 0$

$R = r$, $V = \mathcal{E}/2$

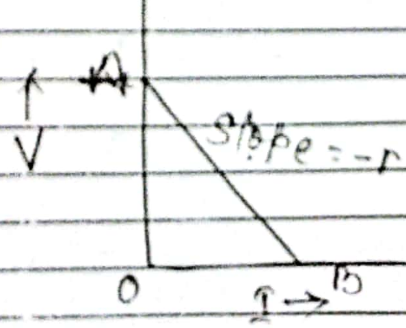
$R \to \infty$, $V = \mathcal{E}$

(vi) The graph between p.d. (V) and current (I) is a straight line -ve slope.

Now, for point A, $I = 0$

So

$$V_A = \mathcal{E}$$



For point B, $V = 0$

$$\therefore \mathcal{E} = I_B r$$

HEATING EFFECTS OF CURRENT

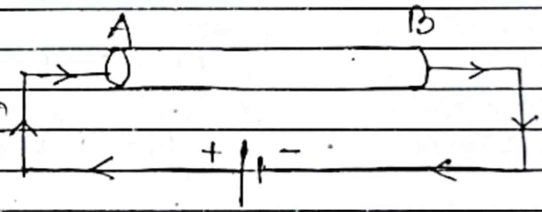
The phenomenon of the production of heat in a resistor by the flow of an electric current through it is called heating effect of current.

Mechanism:–

When a potential difference is applied across the ends of conductor then electric field is setup and the free electrons get accelerated in the opposite direction of the field. This is because during the course of their motion, the electrons collide frequently with the positive metal ions. The K.E. gained by the electrons during the intervals of free acceleration between collision is transferred to the metal ions at the time of collision. The metal ions begin to vibrate about their mean position more and more violently and average K.E increases so increase the temperature of conductor. Thus conductor gets heated due to the flow of current.

Expression for heat developed in a conductor by the passage of an electric current:–

Consider a conducting wire AB of length l, uniform area of cross-section A and consists of n free electrons per unit volume then the resistance of wire is



$$R = \frac{ml}{Ane^2\tau}$$

Let a source of e.m.f. maintains potential difference V between its two ends and sends a steady current I from end A to end B is given by

$$I = v_d A n e \quad \text{——(1)}$$

The amount of charge that flows from one end to another end in t second is

$$q = It$$

As the charge q moves through conducting wire, its potential energy decreases by the amount.

$$U = \text{final P.E at B} - \text{final P.E at A}$$
$$= qV_B - qV_A$$
$$= -q(V_A - V_B)$$
$$= -qV$$

If the charge move through conductor without suffiring collisions, their K.E would charge so that the total energy is unchanged. By the conservation of energy, the charge in K.E must be

$$K = -U = qV = VIt$$

Also when charges move freely through the conductor under the action of electric field, their K.E increase as they move and charge carriers do not move with any acceleration but with steady drift velocity. This is because of the collision of electrons with ions and atoms during the motion. ~~and~~ The K.E gained by the electrons is shared with the metal ions and these ions vibrate more vigorously and conductor gets heated up. The amount of ~~heat~~ energy dissipated as heat in conductor in time t is

$$H = VIt \text{ joules}$$

or $\boxed{H = I^2Rt \text{ joule}}$ $[\because V = IR]$

or, $\boxed{H = \frac{V^2t}{R} \text{ joule}}$ $[\because I = V/R]$

This is the required expression.

Joule's law:– According to this law, the heat produced in a resistor or conductor is

(i) directly proportional to the square of the current for given R.

(ii) directly proportional to the resistance R for a given current (I).

& (iii) directly proportional to the time for which the current flows through resistor.

Note:- 1 Calorie = 4.18 joule $\simeq$ 4.20 joule

$$\therefore H = \frac{VIt}{4.18} \text{ Cal.} = \frac{I^2Rt}{4.18} \text{ Cal.} = \frac{V^2t}{4.18} \text{ Cal.}$$

Electric Power :-

The rate at which work is done by a source of e.m.f. in maintaining an electric current through a circuit is called electric power.

or The rate at which an appliance converts electric energy into other form of energy is called its electric power.

If I current flows through a circuit or conductor for time t second under the constant potential difference of V, then work done to maintain the current is given by

$$W = VIt$$

Therefore electric power,

$$P = W/t$$

$$= \frac{VIt}{t}$$

$$\therefore \boxed{P = VI} \quad \text{——— (i)}$$

But $V = IR$, R is the resistance of a circuit.

$$\therefore \boxed{P = I^2R} \quad \text{——— (ii)}$$

Also $I = V/R$

$$\therefore \boxed{P = V^2/R} \quad \text{——— (iii)}$$

The S.I. unit of electric power is volt × ampere or $(ampere)^2$ ohm or $(volt)^2$ ohm^{-1} or Joule/s or Watt.

1 watt electric power:-

The power of an electric circuit is said to be one watt if one ampere current flows in it against a potential difference of 1 volt.

i.e, 1 Watt = 1 volt × 1 ampere.

Note:-

(i) The electric power supplied by source of e.m.f E is

$$\boxed{P = EI}$$

(ii) The commercial unit of electric power is horse power (H.P.), so, 1 H.P. = 746 watt.

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Electric Energy:— The total work done by the source of e.m.f. in maintaining the current in an electric circuit for a given time is called electric energy consumed in the circuit.

$$\text{Electric energy} = \text{Total work done}$$
$$= VIt$$
$$= Pt$$
$$= I^2Rt$$

The S.I. unit of electric energy is joule or volt × amp × sec. or watt × sec or $(\text{amp})^2$ ohm sec.

Note:— The commercial unit of electric energy is called kilowatt hour (KWh) or Board of trade unit (B.O.T.U) or unit of electricity.

$$1 \text{ KWh} = 1 \text{ kilo watt} \times 1 \text{ hour}$$
$$= 10^3 \text{ watt} \times 1 \text{ hr.}$$

Thus, 1 KWh is the total electric electric energy consumed when an electrical appliance of power 1 kilo watt works for 1 hr.

$$\therefore 1 \text{ KWh} = 10^3 \text{ watt} \times (60 \times 60) \text{ s}$$

$$\boxed{1 \text{ KWh} = 3.6 \times 10^6 \text{ Joule}}$$

Power Rating:— The power rating of an electrical appliance is the electric energy consumed per second by the appliance when connected across the marked voltage of the mains.

5th

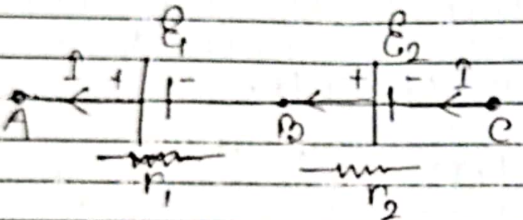
Combination of Cells :-

B.K. Jha

Following are the three ways in which cells are combined to increase the current as well as electric power of a circuit.

1. Series Combination :- The cells are said to be connected in series if negative terminal of one cell is connected to the positive terminal of second cell and so on, such that e.m.f. acts in same direction.

* Consider two cells of e.m.fs $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances r_1 and r_2 are connected in series between the two points A and C. Let I be the current flowing through the series combination is shown in adjoining figure.



Let V_A, V_B and V_C be the potentials at points A, B and C respectively, then potential difference across the terminals of two cells will be

$$V_{AB} = V_A - V_B = \mathcal{E}_1 - Ir_1 \quad \text{--- (i)}$$

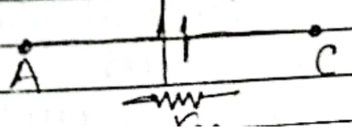
and $$V_{BC} = V_B - V_C = \mathcal{E}_2 - Ir_2 \quad \text{--- (ii)}$$

Thus the potential difference between the terminals A and C of series combination is

$$V_{AC} = V_A - V_C = V_{AB} + V_{BC}$$
$$= (\mathcal{E}_1 - Ir_1) + (\mathcal{E}_2 - Ir_2)$$
$$= (\mathcal{E}_1 + \mathcal{E}_2) - I(r_1 + r_2) \quad \text{--- (iii)}$$

If we wish to replace the series combination by a single cell of e.m.f. $\mathcal{E}_{eq}$ and internal resistance r_{eq}, then

$$V_{AC} = \mathcal{E}_{eq} - Ir_{eq} \quad \text{--- (iv)}$$



Comparing eqs. (iii) & (iv), we get

2

$\boxed{\mathcal{E}_{eq.} = \mathcal{E}_1 + \mathcal{E}_2}$ and $\boxed{r_{eq.} = r_1 + r_2}$

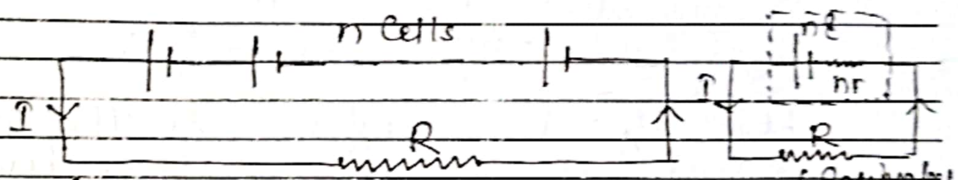
Hence the equivalent e.m.f. of series combination of n cells is equal to the algebric sum of their individual e.m.fs.

i.e. $\mathcal{E}_{eq.} = \mathcal{E}_1 + \mathcal{E}_2 + \dots\dots + \mathcal{E}_n$.

Similarly, the equivalent internal resistance of series combination of n cells is equal to the algebric sum of the individual internal resistance of cells.

i.e. $r_{eq.} = r_1 + r_2 + \dots\dots + r_n$ **

* Consider n identical cells each of e.m.f $\mathcal{E}$ and internal resistance r be connected in series. Let an external resistance R is connected to the combination of cells.



(Series Combination of n Cells) (Equivalent Circuit)

Now, the total e.m.f. of n cells in series
= sum of e.m.f. of all cells
= $\mathcal{E} + \mathcal{E} + \dots\dots$ n cells
= $n\mathcal{E}$

And the total internal resistance of n cells in series
= $r + r + \dots\dots$ n times
= nr

So, the total resistance of the circuit is
= $R + nr$

Therefore, the current in the circuit is

$$I = \frac{\text{Total e.m.f}}{\text{Total resistance}}$$

$$\therefore \boxed{I = \frac{n\mathcal{E}}{R + nr}}$$

Special Cases:-

(i) If the external resistance of the circuit is much greater than the total internal resistance of the cells, then nr can be neglected.

$$\therefore I = \frac{nE}{R}$$

= n times of the current that can be drawn from one cell.

Hence such type of series combination of cells is more useful to increase the current.

(ii) If external resistance of the circuit is much less than total internal resistance of cells, then R can be neglected.

$$\therefore I = \frac{nE}{nr} = \frac{E}{r}$$

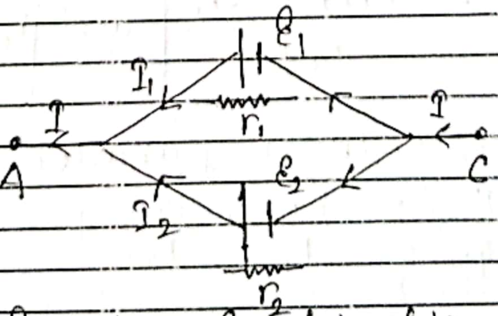
= Current given by one cell.

Hence such type of series combination is not useful to increase the current in a circuit. **

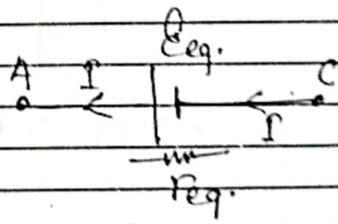
2. Parallel Combination:- The cells are said to be connected in parallel if they are connected in such a way that the positive terminal of all cells are connected at one point and their negative terminals at another point.

* Consider two cells of e.m.fs E_1 and E_2 and internal resistances r_1 and r_2 are connected in parallel between two points. Let I_1 and I_2 are the currents supplied by the two cells then the total current

$$I = I_1 + I_2 \quad \text{——— (1)}$$



(Parallel Combination of two Cells)



(Equivalent Circuit)

As the two cells are connected in parallel between the same two points, so the P.d. V across both cells must be same.

Then

$$V = \mathcal{E}_1 - I_1 r_1$$

$$\therefore I_1 = \frac{\mathcal{E}_1 - V}{r_1} \quad \text{——— (ii)}$$

and

$$V = \mathcal{E}_2 - I_2 r_2$$

$$\therefore I_2 = \frac{\mathcal{E}_2 - V}{r_2} \quad \text{——— (iii)}$$

Therefore,

$$I = \left(\frac{\mathcal{E}_1 - V}{r_1}\right) + \left(\frac{\mathcal{E}_2 - V}{r_2}\right)$$

$$= \left(\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

or, $$V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \left(\frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 r_2}\right) - I$$

or, $$V = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{(r_1 + r_2)} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right) \quad \text{——— (iv)}$$

If we wish to replace the parallel combination by a single cell of e.m.f. $\mathcal{E}_{eq.}$ and internal resistance $r_{eq.}$, then

$$V = \mathcal{E}_{eq.} - I r_{eq.} \quad \text{——— (v)}$$

Comparing eqs. (iv) and (v), we get

$$\boxed{\mathcal{E}_{eq.} = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}} \quad \text{and} \quad \boxed{r_{eq.} = \frac{r_1 r_2}{r_1 + r_2}}$$

Also, for n cells in parallel,

$$\frac{\mathcal{E}_{eq.}}{r_{eq.}} = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} + \cdots \frac{\mathcal{E}_n}{r_n}$$

and $$\frac{1}{r_{eq.}} = \frac{1}{r_1} + \frac{1}{r_2} + \cdots\cdots \frac{1}{r_n}$$

**

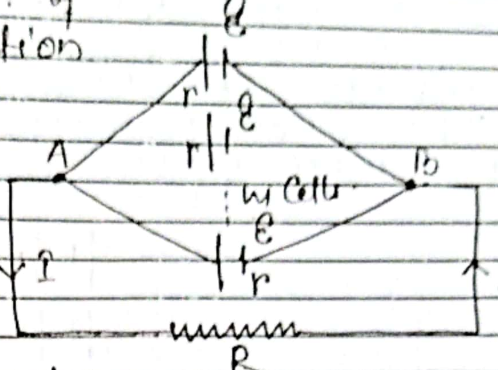
* Consider m identical cells of e.m.f. $\mathcal{E}$ and internal resistance r be connected in parallel between the two points A and B. Let an external resistance R is connected to the combination of cells is shown in adjoining fig.

Now, the total e.m.f. of parallel combination

= e.m.f. due to single cell

$= \mathcal{E}$

And the total internal resistance of m cells are connected in parallel is

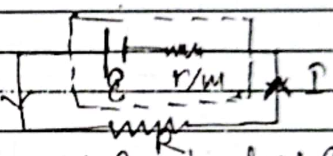


(Parallel Combination of m Cells)

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \text{------ m times}$$

$$= \frac{m}{r}$$

$$\therefore r' = r/m$$



(Equivalent Circuit)

So, the total resistance of the circuit

$= R + r'$

$= R + r/m$

Therefore, the current in the circuit is

$$I = \frac{\text{Total e.m.f.}}{\text{Total resistance}}$$

$$= \frac{\mathcal{E}}{R + r/m}$$

$$\boxed{I = \frac{m\mathcal{E}}{mR + r}}$$

Sp. Cases:- (i) If external resistance of circuit is greater than total internal resistance of the combination of cells, then

$I = \frac{\mathcal{E}}{R}$ = Current given by one cell.

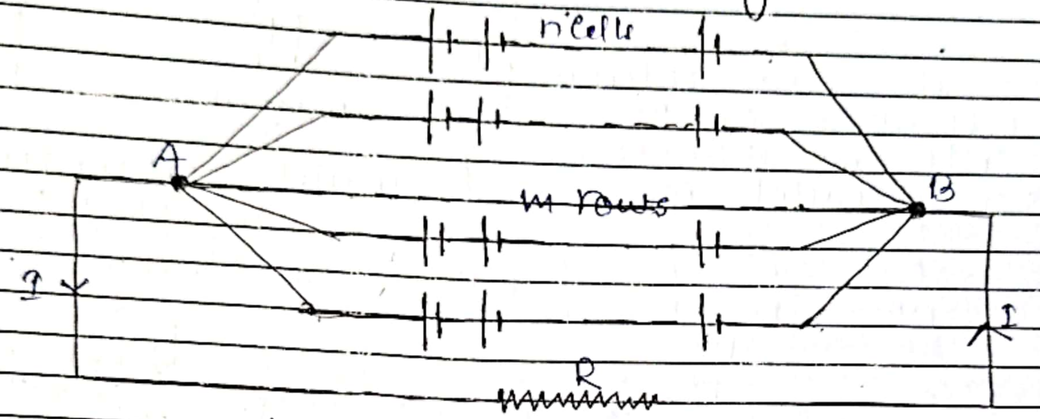
(ii) If $R << r/m$, then

$I = \frac{m\mathcal{E}}{r}$ = m times the current due to single cell.

So such type of combination is useful. **

3. Mixed Combination:— The cells are said to be connected in mixed if number of cells is connected in series in a row and number of such rows is connected in parallel.

Consider n cells each of e.m.f E and internal resistance r are connected in series in a row and m such rows are connected in parallel between the two points A and B. Let the combination is connected with an external resistance R is shown in diagram.



(Mixed grouping of Cells)

Now, the e.m.f. of each row of n cells in series is equal to the total ~~combination~~ e.m.f. of the combination of cells $= nE$

And the internal resistance of each row of n cells $= nr$

So, the total internal resistance of the combination of cells is

$$\frac{1}{r'} = \frac{1}{nr} + \frac{1}{nr} + \ldots\ldots m \text{ times} = \frac{m}{nr}$$

$$\text{or, } r' = \frac{nr}{m}$$

Total resistance of the circuit

$$= R + r'$$

$$= R + \frac{nr}{m}$$

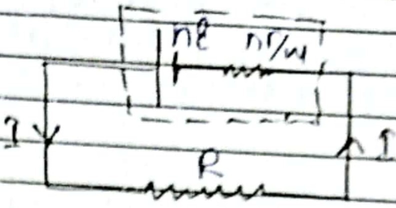
Therefore, current through external resistance R,

$$I = \frac{\text{Total e.m.f}}{\text{Total resistance}}$$

or, $I = \dfrac{n\mathcal{E}}{R + nr/m}$

or, $I = \dfrac{mn\mathcal{E}}{mR + nr}$

or, $\boxed{I = \dfrac{N\mathcal{E}}{mR + nr}}$



(Equivalent Circuit)

[$\because N = m \times n$ is the total number of cells]

Hence the current I will be maximum if denominator i.e. $(mR + nr)$ is minimum.

Now

$$mR + nr = (\sqrt{mR})^2 + (\sqrt{nr})^2$$

$$= (\sqrt{mR} + \sqrt{nr})^2 - 2\sqrt{mR}\sqrt{nr}$$

$$+2\sqrt{mR}\sqrt{nr}$$

$$= (\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnRr}$$

As the perfect square can not be negative, so $mR + nr$ will be minimum if

$$\sqrt{mR} - \sqrt{nr} = 0$$

or, $mR = nr$

or, $\boxed{R = \dfrac{nr}{m}}$

Thus, in a mixed grouping of cells, the current through external resistance will be maximum if the external resistance is equal to the total internal resistance of the combination of cells.

Note:- (i) For a series combination, the reciprocal of the effective power is equal to the sum of the reciprocal of the individual powers

i.e. $\dfrac{1}{P_{eff.}} = \dfrac{1}{P_1} + \dfrac{1}{P_2} + \cdots\cdots$

(ii) In series combination of the bulbs of different powers then $R \propto \frac{1}{P}$. [$\because P = I^2R$]

The bulb of lowest power will have maximum resistance and it will glow with maximum brightness.

(iii) For a parallel combination the effective power is equal to the sum of the powers of the individual

$$\text{i.e., } P_{eff.} = P_1 + P_2 + \ldots$$

(iv) In the parallel combination of the different bulbs,

$$P \propto \frac{1}{R}. \qquad \left[\because P = \frac{V^2}{R}\right]$$

So the resistance of the highest wattage (Power) bulb is minimum, it will glow with maximum brightness.

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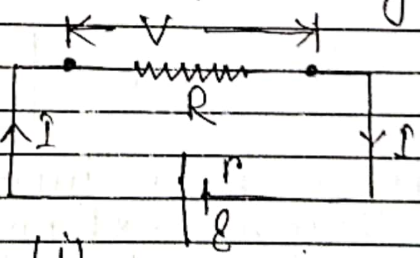
Maximum Power Theorem

It states that the power output across load due to a cell or battery is maximum if the load resistance is equal to the effective internal resistance of cell or battery.

Consider a cell of e.m.f. $\mathcal{E}$ and internal resistance r is connected to an external resistance R, then current through the circuit is



$$I = \frac{\text{Total e.m.f.}}{\text{Total resistance}}$$

$$= \frac{\mathcal{E}}{R+r} \quad \text{——— (1)}$$

The power output of the resistive device will be

$$P = I^2 R$$

$$= \left(\frac{\mathcal{E}}{R+r}\right)^2 R$$

$$= \frac{\mathcal{E}^2 R}{(R+r)^2}$$

or, $P = \dfrac{\mathcal{E}^2 R}{(R-r)^2 + 4Rr}$ —— (ii)

In order to obtain maximum power theorem, we have to equate

$$\frac{dP}{dR} = 0$$

or $\dfrac{d}{dR}\left[\dfrac{\mathcal{E}^2 R}{(R+r)^2}\right] = 0$

or, $\dfrac{d}{dR}\left[\dfrac{R}{(R+r)^2}\right] = 0$

or, $\dfrac{d}{dR}\left[R(R+r)^{-2}\right] = 0$

or, $\left[(R+r)^{-2} + R(-2)(R+r)^{-3}\right] = 0$

or, $(R+r)^{-3}\left[r-R\right] = 0$

As $(R+r)^{-3} \neq 0$, for finite value of R,

So

$r - R = 0$

or, $\boxed{R = r}$

Therefore, the power supplied by the source to the circuit is maximum ($\mathcal{E}^2/4R$) when $R = r$. [* $P_{max} = \mathcal{E}^2/4r$ *]

Note :-

(i) When battery or cell is shorted, R becomes zero, therefore power output is zero. So the entire power of battery or cell is dissipated as heat inside the battery due to its internal resistance, which is

$$P_{diss.} = I^2 r = \left(\frac{\mathcal{E}}{r}\right)^2 r$$

$$= \frac{\mathcal{E}^2}{r}$$

(ii) Efficiency of source of e.m.f. is

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{VI}{\mathcal{E}I} = \frac{IR}{I(R+r)}$$

$$\therefore \boxed{\eta = \frac{R}{R+r}}$$